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| **Grade 9 GLEs not matched by the new high school math standards** | | | **Comment** |
| [9] | E&C-1 | **The student solves problems (including real-world situations) using estimation by** judging whether the strategy will result in an answer greater or less than the exact answer (M3.4.1) |  |
| [9] | E&C-2 | **The student accurately solves problems (including real-world situations) by** adding or subtracting rational numbers including integers with whole number exponents (M3.4.2) |  |
| [9] | E&C-3 | **The student accurately solves problems (including real-world situations) by** multiplying or dividing rational numbers including integers with whole number exponents (M3.4.3) |  |
| [9] | E&C-5 | **The student accurately solves problems (including real-world situations) by** multiplying or dividing numbers in scientific notation (L) (M3.4.3) | New Grade 8 Standard (8.EE.4) |
| [9] | F&R-1 | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** describing or extending patterns (families of functions: linear quadratic, absolute value) up to the nth term, represented in tables, sequences, graphs, or in problem situations (M4.4.1) | New Grade 6, 7 and 8 Standards (6.NS.7; 7.NS.1; 8.F.3) |
| [9] | F&R-3 | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** describing in words how a change in one variable in a formula affects the remaining variables (e.g., how changing the radius affects the volume of a cylinder) (M4.3.2) |  |
| [9] | F&R-4 | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** using a calculator as a tool when describing, extending, representing, or graphing patterns or linear equations (L) (M4.4.2) |  |
| [9] | F&R-5 | **The student demonstrates algebraic thinking by** modeling (graphically or algebraically) or solving situations (including real-world applications) using systems of linear equations (M4.4.3) | New Grade 8 Standard (8.EE.8) |
| [9] | G-1 | **The student demonstrates an understanding of geometric relationships by** identifying, analyzing, comparing, or using properties of angles (including supplementary or complementary) or circles (degrees in a circle) (M5.4.1) | New Grade 7 Standard (7.G.5.) |
| [9] | G-2 | **The student demonstrates conceptual understanding of similarity, congruence, symmetry, or transformations of shapes by** using a coordinate plane to solve problems involving congruent or similar shapes (M5.4.3) | New Grade 8 Standard (8.G.2.) |

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| [9] | | MEA-1 | | **The student demonstrates understanding of measurable attributes by** estimating or converting measurements between the English and metric systems in real-world applications, given a conversion factor (e.g., miles/kilometers) (M2.4.2) |  |
| [9] | | N-1 | | **The student demonstrates understanding of real numbers by** converting between a rational number in scientific notation and standard form (M1.4.4 & M3.4.4) |  |
| [9] | | N-3 | | **The student demonstrates conceptual understanding of mathematical operations by** [using models, explanations, number lines, real-life situations L] describing or illustrating the effects of arithmetic operations on real numbers (M1.4.3) |  |
| [9] | | N-4 | | **The student demonstrates conceptual understanding of mathematical operations by** using models, explanations, number lines, real-life situations, describing, or illustrating the use of inverse operations (squaring/square root) (M1.4.3 & 1.4.5) | New Grade 8 Standard (8.EE.2) |
| [9] | | N-5 | | **The student demonstrates conceptual understanding of number theory by** applying the rules for order of operations to real numbers and variables (M1.3.5) |  |
| [9] | | N-6 | | **The student demonstrates conceptual understanding of number theory by** using distributive property with variables (L) (M1.4.5) | New Grade 8 Standard (8.EE.7) |
| [9] | | S&P-2 | | **The student demonstrates an ability to analyze data (comparing, explaining, interpreting, evaluating, making predictions, describing trends; drawing, formulating, or justifying conclusions) by** using information from a variety of displays or analyzing the validity of statistical conclusions found in the media (M6.4.1) |  |
| [9] | | S&P-3 | | **The student demonstrates an ability to analyze data (comparing, explaining, interpreting, evaluating, making predictions, describing trends; drawing, formulating, or justifying conclusions) by** using range and measures of central tendency to determine the best representation of the data for a practical situation (M6.4.3) | New Grade 7 Standard (7.SP.3) |
| [9] | | S&P-5 | | **The student demonstrates a conceptual understanding of probability and counting techniques by** determining or comparing the experimental and/or theoretical probability of independent or dependent events (M6.4.5) | New Grade 7 Standards (7.SP.8.) |
| [9] | | S&P-6 | | **The student demonstrates a conceptual understanding of probability and counting techniques by** making predictions about the probability of independent or dependent events and using the information to solve problems (M6.4.5) |  |
| [9] | | S&P-7 | | **The student demonstrates a conceptual understanding of probability and counting techniques by** designing, conducting, analyzing, and communicating the results of a probability experiment (L) (M6.4.6) |  |
| [9] | | PS-1 | | **The student demonstrates an ability to problem solve by** selecting, modifying, and applying a variety of problem-solving strategies (e.g., charts, graphing, inductive and deductive reasoning, Venn diagrams) and verifying the results (M7.4.2) | The GLE math process skills are incorporated in to the Standards for Mathematical Practice. The descriptions of the Standards for Mathematical Practice follow this chart as well as the grade-span descriptors appropriate to this grade level. |
| [9] | | PS-2 | | **The student demonstrates an ability to problem solve by** evaluating, interpreting, and justifying solutions to problems by using an alternative strategy (M7.4.3) |
| [9] | | PS-3 | | **The student communicates his or her mathematical thinking by** representing mathematical problems numerically, graphically, and/or symbolically, translating among these alternative representations; or using appropriate vocabulary, symbols, or technology to explain, justify, and defend strategies and solutions (M8.4.1 |
| [9] | | PS-4 | | **The student demonstrates an ability to use logic and reason by** following and evaluating an argument, judging its validity using inductive or deductive reasoning and logic; or making and testing conjectures (M9.4.1 & M9.4.2) |
| [9] | | PS-5 | | **The student demonstrates the ability to apply mathematical skills and processes across the content strands by** using real-world contexts such as science, humanities, peers, community, careers, and national issues (M10.4.1 & M10.4.2) |
| **Grade 10 GLEs not matched by the new high school math standards** | | | | | **Comment** | |
| [10] | E&C-1 | | **The student solves problems (including real-world situations) using estimation by** explaining why one strategy is more appropriate than another and determining why the estimation result is greater or less than the exact answer (L) (M3.4.1) | |  | |
| [10] | E&C-2 | | **The student accurately solves problems (including real-world situations) by** applying basic operations with real numbers using powers [and scientific notation L] (M3.4.2 & M3.4.3) | |  | |
| [10] | E&C-3 | | **The student accurately solves problems (including real-world situations) by** solving problems involving percent increase or decrease (M3.4.5) | | New Grade 7 Standard (7.RP.3.) | |
| [10] | F&R-1 | | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** describing or extending patterns (families of functions: linear, quadratic, absolute value) up to the nth term, represented in tables, sequences, graphs, or in problem situations (M4.4.1) | |  | |
| [10] | F&R-2 | | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** generalizing equations and inequalities (linear, quadratic, absolute value) using a table of ordered pairs or a graph (M4.4.4) | | New Grade 6 Standards (6.EE.5.; 6.EE.7.) | |
| [10] | F&R-3 | | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** describing in words how a change in one variable or constant in an equation affects the outcome of the equation (M4.3.2) | |  | |
| [10] | F&R-4 | | **The student demonstrates conceptual understanding of functions, patterns, or sequences including those represented in real-world situations by** using a calculator as a tool when describing, extending, representing, or graphing patterns, linear, equations, or quadratic equations (L) (M4.4.2) | |  | |
| [10] | G-2 | | **The student demonstrates an understanding of geometric relationships by** using isometric drawings to create 2-dimensional drawings of 3-dimensional objects (shapes that are composites of rectangular right prisms) (L) (M5.4.2) | |  | |
| [10] | MEA-1 | | **The student demonstrates understanding of measurable attributes by** converting square and cubic units within the same system, English or metric, in real-world applications (M2.4.2) | |  | |
| [10] | N-1 | | **The student demonstrates understanding of real numbers by** identifying their subsets (natural, whole, integers, rational, irrational) (M1.4.1) | | New Grade 8 Standard (8.NS.1) | |
| [10] | N-4 | | **The student demonstrates conceptual understanding of mathematical operations by** describing or illustrating the effects of arithmetic operations on real numbers (M1.4.3) | |  | |
| [10] | N-6 | | **The student demonstrates conceptual understanding of mathematical operations by** describing or illustrating [counting and adding in different bases L] (M1.4.2) | |  | |
| [10] | N-7 | | **The student demonstrates conceptual understanding of number theory by** identifying or applying commutative, identity, associative, inverse, or distributive properties to real numbers and variables (M1.4.5) | |  | |
| [10] | N-8 | | **The student demonstrates conceptual understanding of number theory by** identifying or writing the prime factorization of a variable expression using exponents (M1.4.4) | |  | |
| [10] | S&P-7 | | **The student demonstrates a conceptual understanding of probability and counting techniques by** designing, conducting, analyzing, and communicating the results of a multi-stage probability experiment (L) (M6.4.6) | | New Grade 7 Standard (7.SP.8.) | |

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| [10] | PS-1 | **The student demonstrates an ability to problem solve by** applying multi-step, integrated, mathematical problem-solving strategies (M7.4.2) | High School Standards (N-Q.1.,A-SSE.1.)  The GLE math process skills are incorporated in to the Standards for Mathematical Practice. The descriptions of the Standards for Mathematical Practice follow this chart as well as the grade-span descriptors appropriate to this grade level. |
| [10] | PS-2 | **The student demonstrates an ability to problem solve by** verifying the answer by using an alternative strategy (M7.4.3) |
| [10] | PS-3 | **The student communicates his or her mathematical thinking by** representing mathematical problems numerically, graphically, and/ or symbolically, communicating math ideas in writing; or using appropriate vocabulary, symbols, or technology to explain, justify, and defend strategies and solutions (M8.4.1, M8.4.2, & M8. |
| [10] | PS-4 | **The student demonstrates an ability to use logic and reason by** using methods of proof including direct, indirect, and counterexamples to validate conjectures (M9.4.3) |
| [10] | PS-5 | **The student demonstrates the ability to apply mathematical skills and processes across the content strands by** using real-world contexts such as global issues and careers (M10.4.1 & M10.4.2) |

### **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**In grades 9‐12 mathematically proficient students will:**

* make connections between a new problem and previous problems
* determine the question that needs to be answered
* make a plan for attempting a problem
* choose a reasonable strategy
* identify the knowns and unknowns in a problem
* use previous knowledge and skills to simplify and solve problems
* break a problem into manageable parts or simpler problems
* represent algebraic expressions numerically, graphically, concretely/with manipulatives, verbally/written
* explain connections between the multiple representations
* solve a problem in more than one way
* explain the meaning of a problem and look for an entry point
* analyze a problem and make a plan for solving it
* explain correspondence between differing approaches to identify regularity and trends
* check answer using a different method
* identify correspondence between different approaches
* monitor and evaluate progress and change course if necessary
* check the answers to problems using a different method and continually ask, “Does this make sense?”

1. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**In grades 9‐12 mathematically proficient students will:**

* decontextualize to abstract a given situation and represent it symbolically and manipulate the representing symbols.
* reflect during the manipulation process in order to probe into the meanings for the symbols involved
* create a coherent representation of the problem
* make sense of quantities and their relationships in problem situations
* attend to the meanings of quantities
* use flexibility with different properties of operations and objects
* translate an algebraic problem to a real world context
* explain the relationship between the symbolic abstraction and the context of the problem
* compute using different properties
* consider the quantitative values, including units, for the numbers in a problem

**3. Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**In grades 9‐12 mathematically proficient students will:**

* construct arguments using both concrete and abstract explanations
* justify conclusions in a variety of ways, communicate the methodology, and respond to the arguments
* reason inductively about data and make plausible arguments that take into account the context from which the data arose
* understand and use stated assumptions, definitions, and previously established results in constructing arguments
* make conjectures and build a logical progression of statements to explore the truth of the conjectures
* analyze situations by breaking them into cases and recognize and use counter‐examples
* recognize general mathematical truths and statements to justify the conjectures
* identify special cases or counter‐examples that don’t follow the mathematical rules
* infer meaning from data and make arguments using its context
* compare effectiveness of two arguments by identifying and explaining both logical and/or flawed reasoning

**4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two‐way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**In grades 9‐12 mathematically proficient students will:**

* apply mathematics to solve problems in everyday life, society, and workplace
* identify important quantities in a practical situation and map the relationships using such tools as diagrams, two‐way tables, graphs, flowcharts and formulas
* consistently interpret mathematical results in the context of the situation and reflect on whether the results make sense
* apply knowledge, making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later
* make assumptions and approximations to simplify a situation, realizing the final solution will need to be revised
* identify important quantities in a practical situation and map their relationships using such tools as diagrams, two‐way tables, graphs, and formulas
* analyze quantitative relationships to draw conclusions
* improve the model if it has not served its purpose

**5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**In grades 9‐12 mathematically proficient students will:**

* select and accurately use appropriate, available tools (such as pencil and paper, concrete or virtual manipulatives such as geoboards and algebra tiles, graphing and simpler calculators, a spreadsheet, and available technology) when solving a mathematical problem
* identify relevant external and digital mathematical resources and use the resources to pose or solve problems
* detect possible errors by strategically using estimation and other mathematical knowledge
* use technology to visualize the results of varying assumptions, exploring consequences, comparing predictions with data, and deepening understanding of concepts

**6. Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**In grades 9‐12 mathematically proficient students will:**

* communicate precisely to others
* use clear definitions in explanations
* use symbols consistently and appropriately
* specify units of measure, and label axes to clarify the correspondence with quantities in a problem
* calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context
* examine claims and make explicit use of definitions

**7. Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3*(x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

**8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**In all grade levels mathematically proficient students will:**

* discern a pattern or structure
* understand complex structures as single objects or as being composed of several objects
* check if the answer is reasonable

**In all grade levels mathematically proficient students will:**

* identify if calculations or processes are repeated
* use alternative and traditional methods to solve problems
* evaluate the reasonableness of their intermediate results, while attending to the details