Grade 1 Mathematics Standards

Comparison Tool for Standards Transition

Updated March 2012

This document can be used to assist educators in analyzing the commonalities and differences between the new Alaska Mathematics standards and the Fourth Edition (Grade Level Expectations). This document is a first start toward a transition and districts may choose to augment with more detail.

The first column contains the new math standards. The second column shows the Grade Level Expectations (GLEs) that align to the new standards. The third column provides comments, usually highlighting differences between the new standards and GLEs that align in higher grades. Additionally, the comments may include a notation about an increase in rigor. Rigor may be defined as a standard that requires deeper understanding, higher order thinking, expanded analytical processes, or simply a skill introduced at an earlier grade.

Note that some GLEs are coded with an L. This signifies that the GLE was not assessed on the statewide assessment; it was to be assessed at the local level. No new standards are identified as being for local assessment. Students advancing through the grades are expected to meet each year’s grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

In most cases there are not complete matches between the two sets of standards, and it should not be assumed that either the content or skills found in one set of standards will match completely with those of the other set.

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| 6.RP.2. Understand the concept of a unit rate (*a*/*b* associated with a ratio *a:b* with *b ≠*0, and use rate language in the context of a ratio relationship) and apply it to solve real world problems (e.g., unit pricing, constant speed).  *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”* | **[6] E&C-5** developing and interpreting scale models  Any aligned GLE found in the higher grades will need to be absorbed in the lower grade as part of the transition. | Grade 6 GLE provides a specific real world model for understanding unit rate.  **[7] E&C-6** solving proportions using a given scale  **[8] E&C-5** using ratio and proportion |

The new standards represent a shift in the purpose of the standards. They are more instructional in nature, intended to guide classroom curriculum. The new standards do not serve as an assessment document unlike the GLEs. The Department with the support of stakeholders will prepare an assessment framework which will guide the development of the new assessments. The new standards will be assessed starting spring 2016. Until then, all districts will continue administering the Standards Based Assessments aligned to the GLEs through spring 2015.

A table at the end shows the GLEs not matched to the new standards. The comment column indicates where the GLE may be matched to a new standard in a lower or higher grade. Although some GLEs will be taught at other grade levels, teachers must provide opportunities for these GLEs to be reviewed in preparation for the spring Standards Based Assessments through spring 2015.

| **Grade 6 Math GLEs not matched by new standards** | **Comments** |
| --- | --- |
| **The student demonstrates conceptual understanding of fractions (proper or mixed numbers), decimals, percents (whole number), or integers by**  **[6] N-2** identifying place value positions from thousandths to millions (L) (M1.2.2) | 4th and 5th Grade Standards  **(4.NF.6, 4.NF.7, 5.NBT.3)** |

This GLE must be reviewed prior to the SBA through spring 2015.

Finally, the new standards for each grade level define what students should understand and be able to do by the end of each grade. They correspond to Standards of Mathematical Practice. The Standards of Mathematical Practice describe characteristics and traits that mathematics educators at all levels should seek to develop in their students. They describe ways that students should be engaging with mathematics as they progress through school. The integration of these standards into classroom instruction is a key strategy for increasing cognitive demand and conceptual learning. The Standards for mathematical Practice are included at the end of the document.

**Grade 1 Overview**

|  |  |
| --- | --- |
| **Counting, Cardinality, and Ordinality**  • Know ordinal numbers.  • Count to tell the number of objects.  • Compare numbers.  **Operations and Algebraic Thinking**  • Represent and solve problems involving  addition and subtraction.  • Understand and apply properties of operations  and the relationship between addition and  subtraction.  • Add and subtract within 20.  • Work with addition and subtraction equations.  **Number and Operations in Base Ten**  • Extend the counting sequence.  • Understand place value.  • Use place value understanding and properties  of operations to add and subtract.  **Measurement and Data**  • Measure lengths indirectly and by iterating  length units.  • Tell and write time.  • Represent and interpret data.  **Geometry**  • Reason with shapes and their attributes. | **In Grade 1, instructional time should focus on four critical areas:**  (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;  (2) developing understanding of whole number relationships and place value, including grouping in tens and ones;  (3) developing understanding of linear measurement and measuring lengths as iterating length units; and  (4) reasoning about attributes of, and composing and decomposing geometric shapes. |
| **Mathematical Practices (MP)**   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning. |

**Alaska New Mathematics Standards – Counting, Cardinality, and Ordinality**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Counting, Cardinality, and Ordinality 1.CC** |  |  |
| **Skip count and know ordinal names.** |  |  |
| 1.CC.1. Skip count by 2s and 5s. | **[1] N-8** skip counting by 2’s to 20 and 5’s and 10’s to 100 |  |
| 1.CC.2. Use ordinal numbers correctly when identifying object position (e.g., first, second, third, etc.). | **[1] N-3** identifying ordinal position, first to the twentieth |  |
| **Count to tell the number of objects**. |  |  |
| 1.CC.3. Count a large quantity of objects by grouping into 10s and counting by 10s and 1s to find the quantity. | **The student demonstrates conceptual understanding**  • **of whole numbers to one hundred by**  **[1] N-1** reading, writing, ordering/counting and modeling correspondence of whole numbers  **[1] N-8** skip counting by 2’s to 20 and 5’s and 10’s to 100 |  |
| **Compare numbers.** |  |  |
| 1.CC.4. Use the symbols for greater than, less than or equal to when comparing two numbers or groups of objects. | **[1] N-2** comparing whole numbers using the words greater than, less than or equal to | The GLE does not require the use of symbols, which is in the grade 2 GLEs.  **[2] PS-3** translating problems from everyday language into math language and symbols (+, -, =, <, >) |
| 1.CC.5. Order numbers from 1-100. | **The student demonstrates conceptual understanding**  • **of whole numbers to one hundred by**  **[1] N-1** reading, writing, ordering/counting and modeling correspondence of whole numbers |  |
| 1.CC.6. Estimate how many and how much in a given set to 20 and then verify estimate by counting. | **[1] E&C-1** estimating “how many” and “how much” in a given set up to 20 |  |

**Alaska New Mathematics Standards – Operations and Algebraic Thinking**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Operations and Algebraic Thinking 1.OA** |  |  |
| **Represent and solve problems involving addition and subtraction.** |  |  |
| 1.OA.1. Use addition and subtraction strategies to solve word problems (using numbers up to 20), involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using a number line (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem. | **[1] N-6** using objects, pictures, and problem situations to model addition and subtraction of whole numbers  **[1] F&R-3** adding and subtracting whole numbers to 20 using manipulatives to solve story problems | Using unknowns in all positions is a key part of the new standard. Using unknowns is currently addressed in the grade 2 GLEs.  **[2] F&R-3** solving a problem with an unknown (e.g., 7 + ? = 10) |
| 1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g., by using objects, drawings and equations). Record and explain using equation symbols and a symbol for the unknown number to represent the problem. | **[1] F&R-3** adding and subtracting whole numbers to 20 using manipulatives to solve story problems  **[1] F&R-4** creating and solving problems using words, symbols, and drawings | GLE does not address 3 numbers in an addition problem. Symbol for an unknown not addressed in grade 1 (see above). |
| **Understand and apply properties of operations and the relationship between addition and subtraction.** |  |  |
| 1.OA.3. Apply properties of operations as strategies to add and subtract. (Students need not know the name of the property.)  Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition.) Demonstrate that when adding or subtracting zero to any number, the quantity does not change (Identity property of addition.) | **[1] N-10** identifying fact families  **[1] E&C-3** recalling addition and subtraction facts 0-10 | Fact families and addition and subtraction facts are needed for using the associative property, which is not addressed until grade 6. The other two properties are addressed in the grade 2 GLEs below.  **[2] N-5** describing or illustrating the processes of addition and subtraction of whole numbers and their relationships  **[2] N-6** modeling or explaining the commutative and identity properties of addition |
| 1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract 10-8 by finding the number that makes 10 when added to 8.* | **[1] N-10** identifying fact families | Fact families show the inverse relationship between addition and subtraction, which leads to the “unknown-addend” problem. |
| **Add and subtract using numbers up to 20.** |  |  |
| 1.OA.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). | NEW – not addressed in the GLEs |  |
| 1.OA.6. Add and subtract using numbers up to 20. Use strategies such as   * counting on * making ten (8 + 6 = 8 + 2 +4 = 10 + 4 = 14) * decomposing a number leading to a ten (13 - 4 = 13 -3 -1 =10 -1 = 9) * using the relationship between addition and subtraction, such as fact families, (8 + 4 = 12 and 12 - 8 = 4) * creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). | **[1] N-6** using objects, pictures, and problem situations to model addition and subtraction of whole numbers  **[1] N-10** identifying fact families  **[1] E&C-3** recalling addition and subtraction facts 0-10  **[1] E&C-4** recalling doubles to 20 | The new standard identifies specific strategies for adding and subtracting. The GLEs do not expect fluency to sums of 20 until grade 2.  **[2] E&C-4** recalling addition and subtraction facts to 20 |
| **Work with addition and subtraction equations.** |  |  |
| 1.OA.7. Understand the meaning of the equal sign (e.g., read equal sign as “same as”) and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2)* | **[1] F&R-5** using the terms equal to, more than, and less than for numbers up to 20 | The new standard is specific to understanding the meaning of the equal sign and determining true or false. |
| 1.OA.8. Determine the unknown whole number in an addition or subtraction equation. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 6 + 6 = ?, 5 = ? - 3.* | NEW – not addressed in the GLEs | Determining unknowns are not addressed in GLEs until grade 2.  **[2] F&R-3** solving a problem with an unknown (e.g., 7 + ? = 10) |
| **Identify and continue patterns.**  1.OA.9. Identify, continue and label patterns (e.g., aabb, abab). Create patterns using number, shape, size, rhythm or color. | **[1] F&R-1** identifying, naming (e.g., aabb, abab), and continuing a variety of patterns  **[1] F&R-2** creating patterns involving number, shape, size, rhythm, or color |  |

**Alaska New Mathematics Standards – Number and Operations in Base Ten**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Number and Operations in Base Ten 1.NBT** |  |  |
| **Extend the counting sequence.** |  |  |
| 1.NBT.1. Count to 120. In this range, read, write and order numerals and represent a number of objects with a written numeral. | **The student demonstrates conceptual understanding**  • **of whole numbers to one hundred by**  **[1] N-1** reading, writing, ordering/counting and modeling correspondence of whole numbers | The new standard increases upper limit of counting to 120. |
| **Understand place value.** |  |  |
| 1.NBT.2. Model and identify place value positions of two digit numbers. Include:  a. 10 can be thought of as a bundle of ten ones, called a "ten".  b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones.  c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones). | NEW – not addressed in the GLEs | GLEs do not have the equivalent focus on the Number and Operations in Base Ten.  **[2] N-2** modeling and identifying place value positions: ones, tens, and hundreds |
| 1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, <. | **The student demonstrates conceptual understanding**  • **of whole numbers to one hundred by**  **[1] N-1** reading, writing, ordering/counting and modeling correspondence of whole numbers  **[1] N-2** comparing whole numbers using the words greater than, less than or equal to | Grade 1 GLEs do not require the use of symbols for greater than and less than.  **[2] PS-3** translating problems from everyday language into math language and symbols (+, -, =, <, >) |
| **Use place value understanding and properties of operations to add and subtract.** |  |  |
| 1.NBT.4. Add using numbers up to 100 including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10.  Use:   * concrete models or drawings and strategies based on place value * properties of operations * and/or relationship between addition and subtraction;   Relate the strategy to a written method and explain the reasoning used.  Demonstrate in adding two-digit numbers, tens and tens are added, ones and ones are added and sometimes it is necessary to compose a ten from ten ones. | NEW – not addressed in the GLEs | Adding two digit numbers is mentioned in the grade 2 GLEs, but adding multiples of 10 isn’t addressed.  **[2] E&C-5** solving two-digit addition and subtraction problems using a variety of models and algorithms |
| 1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number without having to count. | NEW – not addressed in the GLEs | Skip counting by 10’s and adding or subtracting by 10 is in the grade 2 GLEs.  **[2] N-7** identifying or using patterns in the number system (skip count by 2’s, 5’s, or 10’s; add or subtract by 10; identify even or odd numbers) |
| 1.NBT.6. Subtract multiples of 10 up to 100. Use:   * concrete models or drawings * strategies based on place value * properties of operations * and/or the relationship between addition and subtraction   Relate the strategy to a written method and explain the reasoning used. | NEW – not addressed in the GLEs | The new standard specifies methods for subtracting multiples of 10 up to 100.  **[2] N-7** identifying or using patterns in the number system (skip count by 2’s, 5’s, or 10’s; add or subtract by 10; identify even or odd numbers) |

**Alaska New Mathematics Standards – Measurement and Data**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Measurement and Data 1.MD** |  |  |
| **Measure lengths indirectly and by iterating length units.** |  |  |
| 1.MD.1. Measure and compare three objects using standard or non-standard units. | **[1] MEA-1** measuring and/or comparing objects using standard and nonstandard units |  |
| 1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. | **[1] MEA-1** measuring and/or comparing objects using standard and nonstandard units |  |
| **Tell and write time and work with money.** |  |  |
| 1.MD.3. Tell and write time in hours and half hours using both analog and digital clocks. | **[1] MEA-4** telling time to the nearest half hour using analog and digital clocks |  |
| 1.MD.4. Read a calendar distinguishing yesterday, today and tomorrow. Read and write a date. | **[1] MEA-6** reading a calendar (distinguishing yesterday, today, and tomorrow) |  |
| 1.MD.5. Recognize and read money symbols including $ and ¢. | **[1] MEA-7** recognizing money symbols ($, ¢) |  |
| 1.MD.6. Identify values of coins (e.g., nickel = 5 cents, quarter = 25 cents). Identify equivalent values of coins up to $1 (e.g., 5 pennies = 1 nickel, 5 nickels = 1 quarter). | **[1] MEA-8** identifying equal values of a coin up to a dollar (5 pennies = 1 nickel, 5 nickels = 1 quarter)  [1] MEA-2 identifying money by its value (e.g., penny, nickel, dime, quarter, dollar) | The new standard only includes coins. |
| **Represent and interpret data.** |  |  |
| 1.MD. 7. Organize, represent and interpret data with up to three categories. Ask and answer comparison and quantity questions about the data. | **[1] S&P-1** constructing and using real graphs, pictographs, and bar graphs  **[1] S&P-2** collecting and recording data  **[1] S&P-3** interpreting data with support  **[1] S&P-4** describing information from simple charts/graphs | GLEs require students to collect data. |

**Alaska New Mathematics Standards - Geometry**

| **New Math Standards** | **Grade Level Expectations** | **Comment** |
| --- | --- | --- |
| **Geometry 1.G** |  |  |
| **Reason with shapes and their attributes.** |  |  |
| 1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes. Identify shapes that have non-defining attribute (e.g., color, orientation, overall size). Build and draw shapes given specified attributes. | **[1] G-1** identifying the attributes of 2-dimensional shapes (e.g., a triangle has three sides)  **[1] G-2** identifying and classifying 2 dimensional shapes through visual observations and properties (e.g., which of these shapes is a triangle)  **[1] G-6** drawing, copying, or describing a variety of shapes | Non-defining attributes are covered in kindergarten.  **[K] G-1** sorting and classifying shapes according to similar attributes  **[K] G-2** describing objects using three attributes such as size, color, and shape |
| 1.G.2. Compose (put together) two-dimensional or three-dimensional shapes to form larger (composite) shapes. Compose new shapes from the composite shapes. | NEW – not addressed in the GLEs |  |
| 1.G.3. Partition circles and rectangles into two and four equal parts. Describe the parts using the words*, halves, fourths*, and *quarters* and phrases *half of, fourth of* and *quarter of*. Describe the whole as two of or four of the parts. Understand for these examples that decomposing (break apart) into more equal shares creates smaller shares. | **[1] N-5** dividing geometric shapes into equal halves, fourths, and thirds | The new standard specifies “understand for these examples that decomposing (break apart) into more equal shares creates smaller shares,” which is not in the GLE. |

| **Grade 1 Math GLEs Not Matched by New Standards** | **Comments** |
| --- | --- |
| **The student demonstrates conceptual understanding of simple fractions**  **[1] N-4** dividing an even numbered set of concrete objects (up to 50) into halves | Fractions addressed in new 3rd Grade standards. New 1st grade standard **1.G.3** including halves is related and could be linked to instruction for the standard. |
| **The student demonstrates conceptual understanding of mathematical operations by**  **[1] N-7** identifying groups of objects as repeated addition or equal shares | New 2nd Grade standard **(2.OA.4)** involving repeated addition of objects is related. |
| **The student demonstrates conceptual understanding of number theory by**  **[1] N-9** identifying odd and even numbers up to 20 | New 2nd Grade standard **(2.OA.3)** includes grouping objects to determine is related. |
| **The student demonstrates ability to use measurement techniques by**  **[1] MEA-3** drawing a line segment to the nearest inch | New 2nd Grade standard **(2.MD.3)** |
| **[1] MEA-5** comparing concepts such as: before/after, shorter/longer | New Kindergarten standard **(K.MD.2)** partially addresses comparisons concepts. |
| **The student determines reasonable answers to real-life situations, paper/pencil computations, or calculator results by**  **[1] E&C-2** identifying whether estimation or counting is appropriate with support | New 4th Grade standard **(4.OA.3)** involving determining reasonableness is related. |
| **The student demonstrates an understanding of geometric relationships by**  **[1] G-3** relating real-world examples (e.g., a door is shaped like a rectangle) to the ideas and concepts of geometry | New Kindergarten standard **( K.G.1)**  includes describing objects using names of shapes is related. |
| **The student demonstrates conceptual understanding of similarity, congruence, symmetry, or transformations of shapes by**  **[1] G-4** comparing shapes in the real world | Similarity, congruence and transformations fully addressed in new 8th Grade standards. |
| **The student demonstrates understanding of position and direction by**  **[1] G-5** modeling directional and positional concepts: before, after, between, next to, around, above, below, in the middle of… | New Kindergarten standard **(K.G.1**) partially addresses by only using positional concepts. |
| **The student demonstrates a conceptual understanding of geometric drawings or constructions by**  **[1] G-7** identifying geometric shapes in real-world objects | New Kindergarten and 1st Grade standards **(K.G.5 and 1.G.1)** involving build and draw shapes are related. |
| **The student demonstrates a conceptual understanding of probability and counting techniques by**  **[1] S&P-5** predicting, interpreting, and comparing data using events or repeated observations | Probability addressed in new Grade 6 standards. |
| **The student demonstrates an ability to problem solve by**  **[1] PS-1** creating and solving simple problems using a variety of strategies | The GLE math process skills are incorporated in to the Standards for Mathematical Practice.   1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.     Descriptions of the Standards for Mathematical Practice follow this chart as well as the grade-span descriptors appropriate to this grade level. |
| **The student communicates his or her mathematical thinking by**  **[1] PS-2** translating problems from everyday language into math language and symbols (+, -, =) |
| **[1] PS-3** using everyday language to explain thinking about problem solving strategies and solutions to problems |
| **The student demonstrates an ability to use logic and reason by**  **[1] PS-4** explaining why a prediction or solution is reasonable |
| **[1] PS-5** drawing pictures that support mathematical statements |
| **The student understands and applies mathematical skills and processes across the content strands by**  **[1] PS-6** using real world context (i.e., self, friends, and family) |

**Alaska New Standards for Mathematical Practice**

|  |  |
| --- | --- |
| **1. Make sense of problems and persevere in solving them.**  Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. | **In grades K-2 mathematically proficient students will:**   1. focus on the problem and check for alternate methods 2. check if the solution makes sense |
| **2. Reason abstractly and quantitatively.**  Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. | **In grades K-2 mathematically proficient students will:**   1. represent a situation symbolically and/or with manipulatives 2. create a coherent representation of the problem 3. use units of measurement consistently |
| **3. Construct viable arguments and critique the reasoning of others.**  Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. | **In grades K-2 mathematically proficient students will:**   1. construct arguments using concrete referents such as objects, drawings, diagrams, and actions 2. justify conclusions, communicate conclusions 3. listen to arguments and decide whether the arguments make sense |
| **4. Model with mathematics.**  Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two‐way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. | **In grades K-2 mathematically proficient students will:**   1. apply mathematics to solve problems in everyday life 2. identify important quantities in a practical situation and model the situation with manipulatives or pictures 3. interpret mathematical results in the context of the situation and reflect on whether the results make sense |
| **5. Use appropriate tools strategically.**  Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such  as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. | **In grades K-2 mathematically proficient students will:**   1. select the available tools (such as pencil and paper, manipulatives, rulers, and available technology) when solving a mathematical problem 2. be familiar with tools appropriate for the grade level to make sound decisions about when each of these tools might be helpful 3. identify relevant external mathematical resources and use them to pose or solve problems 4. use technological tools to explore and deepen their understanding of concepts |
| **6. Attend to precision.**  Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. | **In grades K-2 mathematically proficient students will:**   1. give thoughtful explanations to each other 2. use clear definitions and reasoning in discussion with others 3. state the meaning of symbols they choose, including using the equal sign consistently and appropriately |
| **7. Look for and make use of structure.**  Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3*(x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. | **In all grade levels mathematically proficient students will:**   * discern a pattern or structure * understand complex structures as single objects or as being composed of several objects * check if the answer is reasonable |
| **8. Look for and express regularity in repeated reasoning.**  Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. | **In all grade levels mathematically proficient students will:**   * identify if calculations or processes are repeated * use alternative and traditional methods to solve problems * evaluate the reasonableness of their intermediate results, while attending to the details |